Eurex research

EURO STOXX 50® Index implied repo trading at Eurex

Stuart Heath, Director,
Eurex Equity & Index R&D

Stuart Heath

Stuart was Head the Eurex/Deutsche Börse London representative from 2010 to 2016 having spent the previous three years working on product development at Eurex and laterly designing and managing listed dividend futures and options products which were launched in 2008. Prior to joining Eurex Stuart held positions in credit and fixed income trading, covering both credit default swaps and corporate bonds as well as in risk management. Stuart has over 15 years of experience in the finance and holds an MSc in International Banking and a BA(Hons) in Accountancy and Finance.

Introduction

Listed solution for implied equity repo trading via EURO STOXX 50® Index Total Return Futures complement the Eurex suite of equity index derivatives and support the market in complying with new financial market legislation. Eurex Total Return Futures are designed to offer listed solutions for trading the implied equity repo rate. Index TRFs aim to replicate the payoff on an index total return swaps (TRS) in a cost efficient way. This research paper focuses on the inseparable relationship between implied repo rates and equity index total return swaps. Written by Stuart Heath, Director Equity & Index R&D at Eurex, it covers the various aspects and calculations of both repo rates and the TRS.

1. Basic building blocks

Index forward contract: is a contract between two counter-parties, where one party agrees to buy the underlying equity index from the other party for a given price ("index forward price") at a given date in the future ("maturity").

Index forward price: is the price of an index forward contract so that the value of the contract is worth zero today. The index forward price is the market implied expectation of where the spot price of the underlying index will be at maturity.

a. Simple calculation of the index forward price

The basic calculation of the index forward price is relatively straightforward given the current index spot level and relevant annualized funding rate for the period.

Example 1: Index forward price calculation (assuming no income)
The basic index forward price of an equity index (that distributes no income) is equal to:

Index forward price = Index spot × (1 + Funding rate × T)
Where T = Time to maturity (in years)

Assume that index spot is 1,000, funding rate is 4% p.a. and maturity is 3 months i.e. T = 0.25 years, then
the index forward price = 1,000 × (1 + (0.04 ×0.25) = 1,010

Validation:
If the index forward price is below 1,010, for example at 1,005 there is an arbitrage opportunity i.e.:

Buy index forward contract at 1,005 and simultaneously;
Sell index spot at 1,000 and invest the 1,000 proceeds at the 4% rate.

At maturity:
Receive index (from index forward contract) for 1,005 and simultaneously;
Receive a total of 1,010 from investment of index spot proceeds.
This results therefore in a risk-free profit of 5.

Similarly if the index forward price is above 1,010, for example at 1,020, then there is also an arbitrage opportunity i.e.:

Sell index forward contract at 1,020 and simultaneously;
Borrow 1,000 at 4% and buy the spot index at 1,000.

At maturity:
Deliver the index through the index forward contract and receive 1,020;
Pay the loan plus interest total of 1,010.
This results in a risk-free profit of 10.

Forward pricing: the key to pricing index forward contracts is that there are always two alternative investment strategies:

1. Trading the index forward contract at the index forward price
2. Replicating the index by borrowing cash and purchasing the spot index i.e. the basket of underlying component stocks and holding to maturity (or alternatively selling the Index basket of underlying component stocks and investing the cash).

b. Calculation of the index forward price with income

During the course of a payment period there are normally additional adjustments to be made to the calculation based upon the dividends payable to holders of the physical stock of the index and the repo charges associated with stock lending which impact the total financing cost. Hence the calculation has to be adjusted to incorporate these elements.

Example 2: Index forward price calculation with income adjustments

Using Example 1 above but with the additional assumptions:

1. Index pays a dividend at maturity.
2. Index repo is the fee earned by the holder of the cash index that reduces the overall financing cost – and is applied to the funding rate used in the calculation.

Now the forward price of the equity index is equal to:

Index forward price =
Index spot × (1 + (Funding rate – Repo) × T) – Dividend

Assume that index spot is 1,000, funding rate is 4% p.a., maturity T is 0.25, index dividends are 20 index points and the repo rate is 1% p.a.
In this case the index forward is equal to
$1{,}000 \times (1 + (0.04 - 0.01) \times 0.25) - 20 = 987.5$

Validation:
If in trading the index forward price is above 987.5, for example 1,000, then there is an arbitrage opportunity i.e.:

Sell the index forward contract at 1,000 and;
Simultaneously borrow 1,000 at 4% to buy the spot index and;
Lend the spot index to earn the repo fee of 1% p.a.

At maturity:
Receive the dividend of 20 index points and;
Take return of the index, receiving the repo fees of 2.5
Sell back the index through the index forward contract and receive 1,000
Use proceeds to repay the loan at 1,010 (i.e. 1,000 + interest of 4% × 0.25)
This results in a risk free profit of 20 + 2.5 +1,000 – 1010 i.e. 12.5

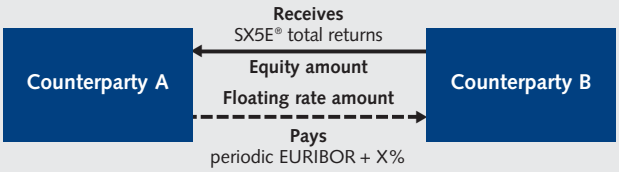
The basic elements used above, namely index spot price, interest rates plus income adjustments for dividends and repo rates, form the basic building blocks for pricing both forwards and futures.

2. Product structure for an equity index total return swap (TRS)

a. Basic structure of an equity index TRS and cash-flows

The basic TRS structure is relatively straightforward. An equity index total return swap is where the buyer and the seller of the contract agree to exchange, at periodic dates, two cash flows based on a notional amount of the reference equity index. Below is a simple example for a TRS based upon the EURO STOXX 50® (SX5E) Index.

Diagram 1



The **equity amount** reflects the P&L of a long position in the underlying index. The equity amount is the total return performance of the index i.e. in case of a price return index such as the SX5E it is the sum of the price return performance plus distributions (such as dividends) – in index points receivable during the period.

The floating rate amount reflects financing cost of the underlying index. The floating amount is quoted as a floating interest rate plus/minus a fixed spread. The floating interest rate is typically based on a benchmark reference rate such as EURIBOR or EONIA.

In the above trade – counterparty A agrees to receive ("buys") the total returns on the SX5E index from counterparty B against paying a floating periodic financing cost of EURIBOR plus/minus a percentage spread of X. Note the fixed spread is set at the swap's inception and the notional value of the swap is not exchanged.

b. Calculation of periodic cashflows

A total return swap on the EURO STOXX 50® would normally exchange cashflows quarterly and the reference floating interest rate used equates to this period i.e. 3month EURIBOR.

One standard often adopted is to make these quarterly payments related to the third Friday of each standard quarterly expiry (Mar, Jun, Sep, Dec cycle) of the EURO STOXX 50® Index Futures listed at Eurex. The calculation of the amounts for each period is as follows:

Example 3: A single period total return swap
At the beginning of the period the equity reference and the floating rate are fixed:

Equity reference ($Spot_0$) is the spot of the equity index as observed at the beginning of the payment period t=0. **Floating rate** ($Euribor_0$) is the EURIBOR as observed at the beginning of the accrual payment period t=0. **Dividends** are those incurred over the accrual payment period from time t=0 to t=1 and calculated as index point equivalent.

At the end of the accrual payment period t=1, the spot of the equity index is recorded ($Spot_{final}$) and the **equity amount** and the **floating rate amount** are calculated:

The **equity amount** is calculated as:

$$\text{Equity amount} = \text{Multiplier} \times \left(\text{Spot}_{final} - \text{Spot}_0 + \sum_{t_0 t_1} \text{Dividends}\right)$$

Where Multiplier = Number of index "baskets" that determine the notional value.

If the equity amount is positive, the buyer receives the amount from the seller. If the equity amount is negative the buyer is obliged to pay the value.

The **floating rate amount** is calculated as:

$$\text{Floating rate amount} = \text{Multiplier} \times \text{Spot}_0 \times \left(\text{Euribor}_0 +/- \text{Spread}\right) \times \frac{N}{360}$$

N is the total number of days in the respective accrual payment period (i.e. total number of business days between t=0 and t=1).

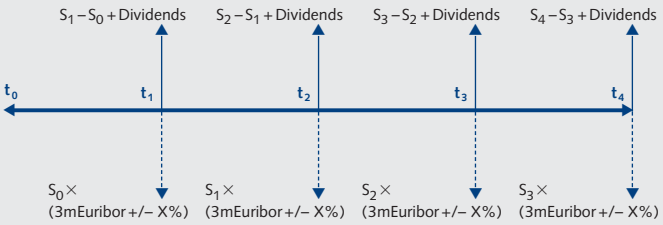
The floating rate amount is paid by the buyer to the seller, though in practice these cashflows are netted.

c. Calculation of multi-period cashflows

The majority of TRS are usually traded over more than one period and hence have more than one periodic date where the relevant cashflows for the preceding period are exchanged and where the reference points for the equity and floating rate are reset for the following period.

For example a one-year TRS on the EURO STOXX 50® (SX5E) with quarterly payments, such as those made on the third Friday expiry dates as mentioned above, would see the following cashflows:

Diagram 2



Where:
S_0 = the value of the spot SX5E at time t_0, S_1 at time t_1 etc. dividends at time t_1 being those paid by the index constituents in the period t_0 to t_1 etc.
The 3m Euribor rate used to calculate cashflow at t_1 is set at the beginning of the period i.e. t_0 etc.

Therefore the quarterly equity amount for counterparty A equates to receipt (or payment if negative) of the change in value of SX5E spot from the beginning of the quarter to end of the quarter, plus any dividends during that period.

On the floating rate amount counterparty A pays 3 month Euribor + X% on the equivalent nominal based on the spot value of SX5E at the beginning of each period. **As a key point it is important to note the size (in nominal terms) of the "basket" is reset at each quarterly date and is dependent on the SX5E spot value at that point.**

Thus at each cashflow dates the following payments will be due.

Period	Equity amount	Floating rate amount
0,25	$\left(\text{Spot}_1 - \text{Spot}_0 + \sum_{(t_0,t_1)} \text{Dividends}\right)$	$\text{Spot}_0 \times \left(\text{Euribor}_0 +/- \text{ Spread}\right) \times \frac{\text{Act}\ (t_0,t_1)}{360}$
0.50	$\left(\text{Spot}_2 - \text{Spot}_1 + \sum_{(t_1,t_2)} \text{Dividends}\right)$	$\text{Spot}_1 \times \left(\text{Euribor}_1 +/- \text{ Spread}\right) \times \frac{\text{Act}\ (t_1,t_2)}{360}$
0.75	$\left(\text{Spot}_3 - \text{Spot}_2 + \sum_{(t_2,t_3)} \text{Dividends}\right)$	$\text{Spot}_2 \times \left(\text{Euribor}_2 +/- \text{ Spread}\right) \times \frac{\text{Act}\ (t_2,t_3)}{360}$
1.0	$\left(\text{Spot}_4 - \text{Spot}_3 + \sum_{(t_3,t_4)} \text{Dividends}\right)$	$\text{Spot}_3 \times \left(\text{Euribor}_3 +/- \text{ Spread}\right) \times \frac{\text{Act}\ (t_3,t_4)}{360}$

The netted amount would then be paid by or to the relevant counterparty at each cashflow date.

3. Equity index total return swap pricing and repo rate

Equity index TRS are "priced" by reference to the spread in relation to the benchmark funding rate, which is fixed at the inception of each TRS. One of the key determinants used to fix the spread is the repo rate. The determination of that repo rate is based on a number of market factors such as:

Short term repos – one of the main drivers of the costs of short-term repos is the inventory of stocks available to be lent. When borrowing demand is high and inventory (available for borrowing) is low, repo fees will tend to rise. High borrowing demand can be due to a number of reasons, such as:

Strong negative outlook on equities means that speculators want to sell the stocks to benefit from the stock going down – but of course they need to borrow it to ensure they can deliver it.

Corporate actions – can lead in some cases to a possible arbitrage and arbitrageurs would want to benefit from it without taking exposure on the stock, hence they would buy the stock and sell a forward, again putting pressure on the repo rate.

However, there are other costs that can be incorporated into the traded spread such as:

Withholding tax – as seen a key element to forward pricing is distributions applied during the reference period. In the case of dividends however these are generally subject to a domestic withholding tax and hence allowances may be made for this.

Balance sheet costs – for a seller of an equity index TRS the immediate hedge could be to buy the cash basket in order to replicate the returns – which may have balance sheet costs that would be passed on through inclusion in the spread.

Frictional costs – a catch all for items such as brokerage commissions which impact cash basket replication etc.

a. Relation between equity index TRS spread and repo rate

In determination of the forward price the repo rate is subtracted from the interest rate to determine the total financing cost i.e. this amount is the assumed income a holder of the cash index will earn in the repo market.

In respect of the equity index TRS the spread is the rate over the reference interest rate that a buyer (receiver) of total returns must pay to the seller (payer) and of which repo rate is the key driver. If the holder of a cash basket can receive income from repo of the underlying stocks then equally the buyer (receiver) of an equity index TRS would expect the spread to be subtracted from the financing cost.

Hence TRS spread is (to a greater part) the inverse of the repo rate.

b. Negative repo rates

In theory, with all things being equal, negative repo rates should be arbitraged away by a simple trade of selling the forwards/futures and buying the cash basket at the current funding rate. The trader can then make risk free profit between the actual repo amount earned and the negative repo rate implied from the forward price (assuming the minimum earnable is zero – but that still implies a profit).

A sustained negative repo rate implies that, instead of earning repo from the cash securities held, the dealer is in fact "paying" an amount to remove the securities from their balance sheet.

This has in fact been the case since 2013. The key driver is the balance sheet constraints currently applicable to banks under the Basel III reforms introduced by the Bank of International Settlements (BIS) and its implementation in Europe under the Capital Requirements Directive IV (CRD IV).

The key impact of this directive is to restrict the traditional financing activities of banks Delta 1 desks as the cost of capital applied to balance sheet use (such as holding cash equities) has made these trades unprofitable. Thus since 2013 negative repo rates have been sustained and equally equity index TRS spreads have stayed positive.

4. Implied repo

In terms of EURO STOXX 50® (SX5E) forward pricing the majority of inputs are directly observable. In particular index spot rate and interest rates are directly observable and distributions such as gross dividends are either directly observable, or if projected, can be hedged with for example the EURO STOXX 50® Index Dividend Futures.

The "other" factors in the determinant of forward prices (predominately the repo rate) are usually implied for market prices and are aggregated under the term of "implied repo".

a. Calculation of implied repo

The shorter end of the implied repo curve is determined using liquid index futures such as the futures on the EURO STOXX 50® Index.

Example 4: Calculation of implied repo
Taking our original index forward price model and applying this to index futures:

$$\text{Forward (futures)} = \text{Spot} \times (1 + (\text{Rate} - \text{Repo}) \times T) - \text{Dividend}$$

EURO STOXX 50® Index Futures:	**3,014.00**
EURO STOXX 50® Index (SX5E) spot:	**3,025.22**
Dividends to maturity	**10.20**
Time to maturity is 90 days	**(T) = 0.25**
Interest rate	**–0.300%**

$$\text{Forward} + \text{Dividend} - \text{Spot} \times (1 + \text{Rate} \times T) = \text{Spot} \times ((-\text{Repo}) \times T)$$

$$3{,}014 + 10.20 - 3{,}025.22 \times (1 + -0.300\% \times 0.25) = 3{,}025.22 \times ((-\text{Repo}) \times 0.25)$$

Therefore calculating out for the implied repo we get –0.165% or –16.5 basis points.

We can therefore determine that of the 11.22 index points of basis (cash index – futures price) of the Dec 16 contract, 1.25 index points is due to the implied repo – the rest being a function of dividends and the interest rate funding.

b. Impact of implied repo

The impact of negative implied repo rates has a knock on effect to structured trading and exotic desks – particularly in Europe. These desks in effect sell quantities structured retail products such as autocallables[1] through which they are selling downside risk. These desks are therefore typically short forward exposure.

Short forward exposure brings two additional risk elements. The dealer will be long dividend exposure (a fall in dividends = a rise in forward prices) and long repo exposure (a fall in repo = a rise in the overall financing costs = a rise in forward prices). Whilst dividend risk can be hedged effectively on the SX5E using EURO STOXX 50® Index Dividend Futures, the negative implied repo represents a cost to trading.

Typically the forward exposure is hedged using a combination of standard index futures and synthetics (call minus put at same strikes in longer dated options), or by using total return swaps (TRS).

5. Index total return futures – Implied repo

Index total return futures represent the final piece in terms of hedging forward exposure by allowing trades to be based on implied repo rates.

A buyer of an index total return future, as with a TRS, receives the total returns of the reference index, the equity amount i.e. both capital and distributions. Against this the buyer pays the financing costs in the form of a funding rate plus or minus a spread. The TRF futures unlike conventional futures are priced in terms of the spread in basis points.

a. Payout profile of an index total return futures (TRFs)

Index TRFs are structured to replicate the net payout profile of the index total returns. In terms of equity amounts the buyer would receive the gross total returns relating to the reference index.

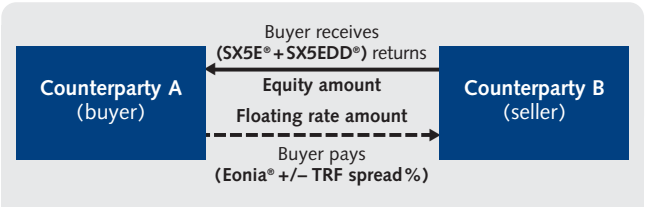
For the EURO STOXX 50® Index (SX5E – a price return index) for example, a buyer would receive the total returns from the SX5E plus any distributions attributable. In the case of the EURO STOXX 50® an additional EURO STOXX 50® Distributions Point Index (SX5EDD) is calculated and the equity amount of the total returns is the sum of these two indexes.

In respect of the financing costs this will be made up of the funding cost attributable to the benchmark funding rate – which for the EUR-denominated SX5E will be EONIA – and the additional spread agreed at the inception of the trade.

For the TRFs the benchmark rate will be used to calculate the funding charge for the index and will be netted with the equity amount (i.e. for the buyer will be subtracted from the equity amounts – representing an underperformance due to these costs[2]).

The remaining spread amount will be determined or traded by agreement between the two parties. This will be priced in basis points on an annualized basis and represents the additional cost (if spread is positive) to be paid by the buyer to the seller of equity index returns until maturity.

Diagram 3 – Outline of total return futures on SX5E



[1] Autocallable is a feature of an exotic option that is often found in structured products with longer maturities. A product with an autocallable feature would be called prior to maturity if the reference index is a predetermined index level on specified observation dates. The investor would receive the principal amount of their investment plus a pre-determined premium (or a coupon) and the product terminates.

[2] The term "costs" in this instance is based on a premise of positive interest rates – in a negative interest rate environment this will be inverted and will be positive in terms of performance.

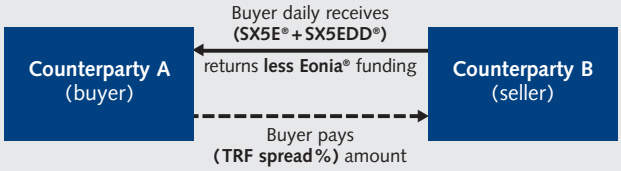
b. Pricing a total return future (TRF)

Theoretically in order to pay the total returns to the buyer, the seller would purchase the cash basket and would have to pay the financing costs to maturity – these charges they would naturally pass on to the buyer.

The financing costs consists of the benchmark interest rate funding charge (i.e. the borrowing cost for the cash used to purchase the cash basket) less any (positive) implied repo that can be achieved (all other costs are disregarded).

In respect of an index total return future the cost related to the benchmark funding rate (EONIA® in the case of SX5E) is incorporated into the daily returns calculation as an underperformance of the equity amount based on the funding amount due.

Diagram 4 – Daily amounts of index total return futures on SX5E



Therefore the outstanding element to be priced is the (implied) repo amount for that index until expiry. A negative repo rate means that financing costs will be greater than the benchmark funding rate as it represents an additional cost and hence the seller would expect the buyer to compensate them for this additional amount. Therefore negative implied repo (which represents an additional cost to the holder of long equity) = positive TRF spread.

Therefore a seller would charge an additional spread to the buyer and this is reflected in index total return futures on the EURO STOXX 50® at Eurex which, in common with market convention, is priced in basis points (one basis point = 0.0001 or 0.01%).

Example 5: Forward prices and total return futures spread
Using a basic application of the traded TRF spread in its use to calculate forward (futures) prices

TRF traded spread in basis points (bps) per annum	**16.5 bps**
EURO STOXX 50® Index (SX5E) spot:	**3,025.22**
Time to maturity is 90 days	**(T) = 0.25**

Therefore the seller requires an additional 16.5 bps to expiry to pay total returns on the SX5E. In terms of index points this is directly calculated as:

Spot × (TRF Spread × T) = 3,025.22 × (0.165% × 0.25)

Therefore calculating out we can see that the seller would require an additional 1.25 index points to cover his implied repo costs.

Now taking the standard forward calculation but excluding repo:

Forward (futures) = Spot × (1 + Rate × T) – Dividend

Expected dividends to maturity	**10.20**
Interest Rate (EONIA® curve) to expiry	**–0.300%**

Forward (futures) = 3,025.22 × (1 + –0.300% × 0.25) –10.2 = 3,012.75

However we know that the seller of the forward (future) in respect of the TRF would demand a premium of 1.25 index points therefore the forward price required by the seller is **3,012.75 + 1.25** index points i.e. forward (futures) price will be **3,014**.

Hence the key driver in pricing the TRF spread required for any maturity is the implied repo associated with carrying the index to that term of expiry.

6. Trading implied repo and forward repo with total return futures

Eurex Index Total Return Futures (TRF) allow traders for the first time to hedge longer term implied repo. The EURO STOXX 50® Index Total Return Futures will offer at least 5 years of quarterly expiring contracts. For an exotic or structured product desk with a short forward exposure, buying the TRF will hedge both the forward exposure and implied repo.

The TRF contracts can additionally be used to hedge or trade forward implied repo associated with longer dated structured products. For example selling a five year TRF and simultaneously buying the one-year TRF expiry in the same amount – will result in a net position of selling implied repo for four years – one year forward. In this case all of the other returns of these legs (equity index, distributions, and EONIA funding) will cancel out of the first year.

For further information please contact

Product Research & Development Equity & Index
Stuart Heath **T** +44-20-78 62-72 53
stuart.heath@eurexchange.com

Global Equity & Index Sales
Nicolas von Kageneck **T** +33-1-55 27-67 69
nicolas.kageneck@eurexchange.com

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Eurex Frankfurt AG
Mergenthalerallee 61
65760 Eschborn
Germany

Eurex Zürich AG
Manessestrasse 85
8045 Zurich
Switzerland

www.eurexchange.com

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